

{curves}

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why?

- Vector graphics are really bad at rendering curves
 - We can't even draw straight lines any more
- Curves can model physical motion in perfect systems
 - Simulations with time steps are better
 - e.g. model dynamic wind

fake it

- Really smooth paths for non-interactive camera motion
- Other unrealistic but smooth motion
 - cartoons
 - scene transitions
- Guiding a 3d model of a surface
 - sea waves
 - visualising a 2d or 3d mathematical function (Matlab)
 - **tessellating** smooth surfaces with control **patches**

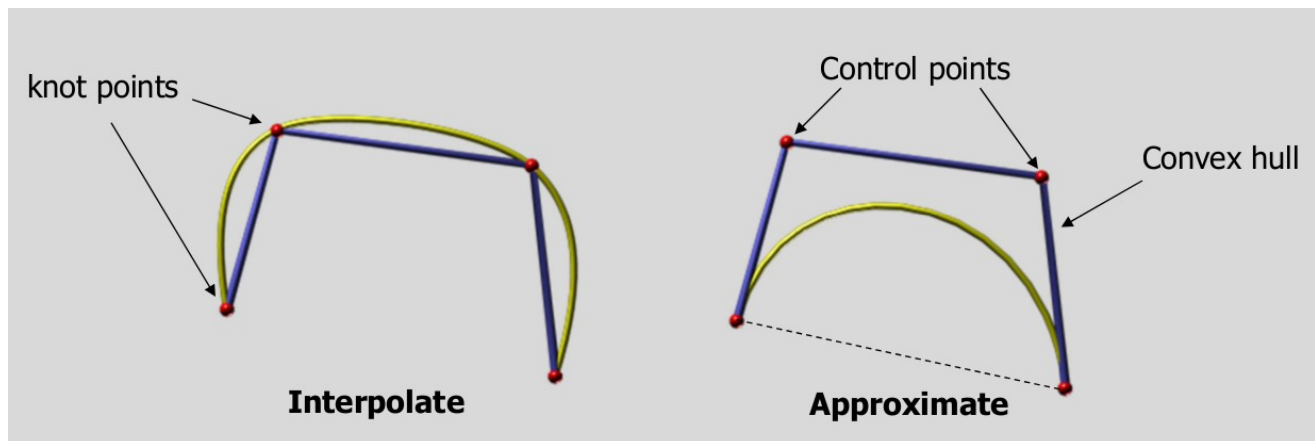
Parametric Curves

- Mathematical function for a curve
- Curve is defined by some parameters

$$x = \sin (t)$$

neat tricks

- sine / cosine are great for a lot of circular curves
- provide just a few **control points** or “**knots**”
 - generate a curve to fit through all the points
 - generate a curve within the points

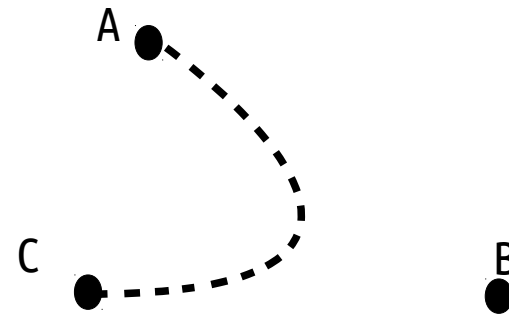


Bézier Curves

- 1962 Pierre Bézier at Renault – popularised Bernstein polynomials in automotive design
- approximate curve
- quite easy to do
- simplest reference: Superbible 6th ed. chapter 4

Bézier Curves

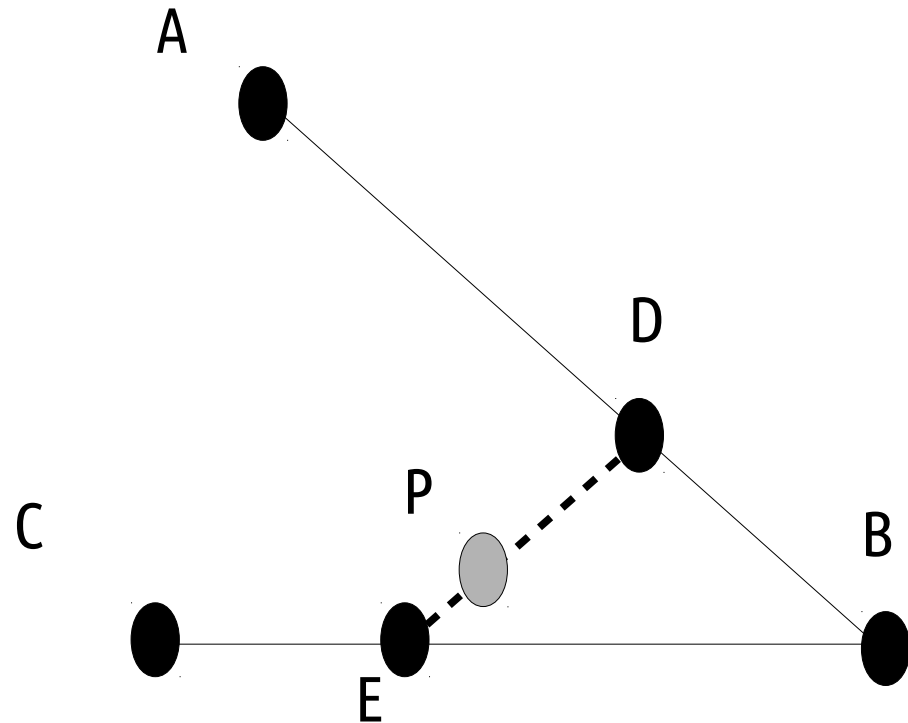
- Define 3 control points
 - A (start)
 - B (top)
 - C (end)
- Set some factor “ t ” 0..1
- Write a little function that returns a point P, given A, B, C, t



Bézier Curves example

- I. $t = 0.666$ or $2/3$
- II. $D = A + 2/3$ of $B - A$
- III. $E = B + 2/3$ of $C - B$
- IV. $P = D + 2/3$ of $E - D$

these are just linear
interpolations



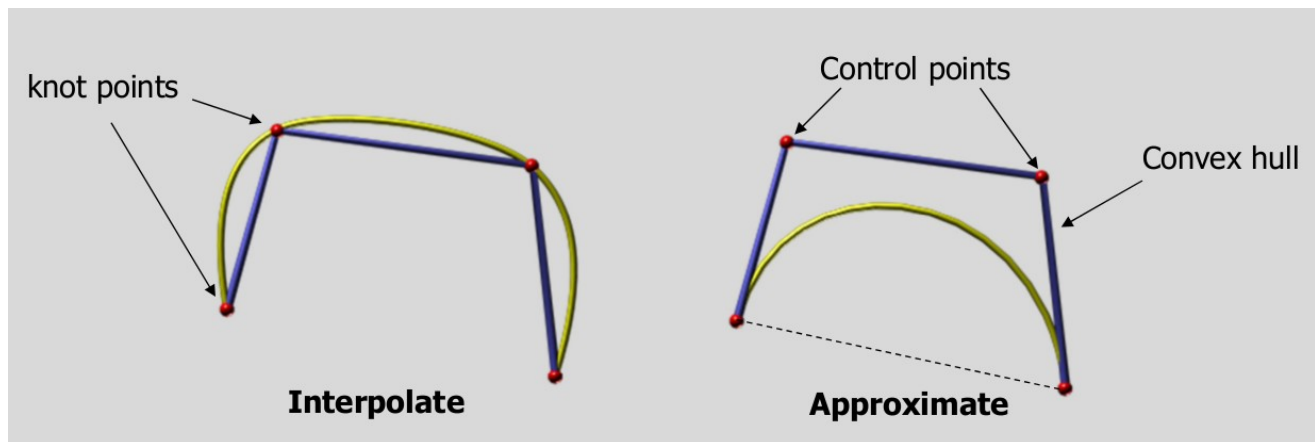
Bézier Curves – Vertex Shader

```
14 uniform float t;
15 uniform vec3 A;
16 uniform vec3 B;
17 uniform vec3 C;
18
19 vec3 quadratic_bezier() {
20     → vec3 D = mix(A, B, t); // D = A + t(B - A)
21     → vec3 E = mix(B, C, t); // E = B + t(C - B)
22     → vec3 P = mix(D, E, t); // P = D + t(E - D)
23     →
24     → return P;
25 }
```

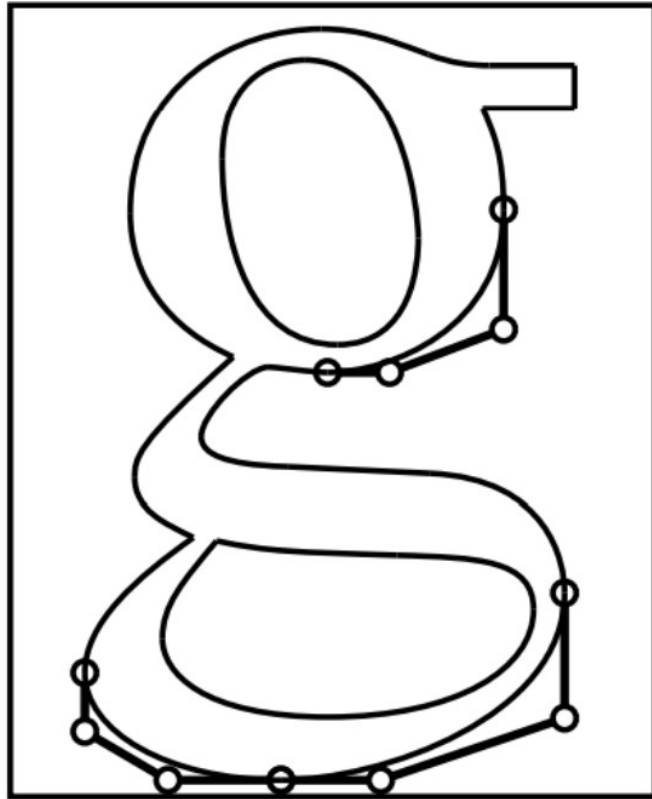
REMIND ME: Show Demo Now

Bézier Curves

- Can add more control points to get higher-order curves
 - More interpolations
- Can add second parameter to get 3d **Bézier surface**...
- **Q. Which type of curve are Bézier?**
- **Q. Motion path problem related to “t”?**



Bézier Curves in Vector Graphics



Font definition using Bézier curves.

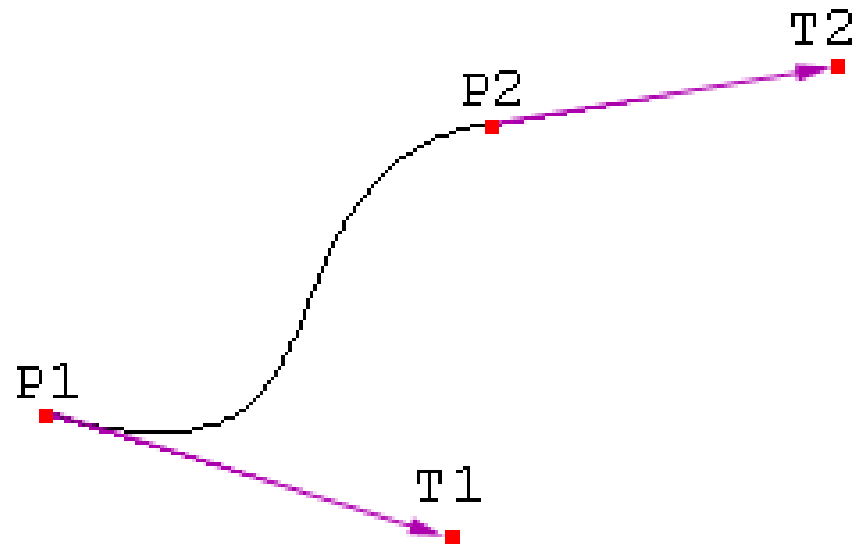
- Font bitmaps don't resize well
- Bézier rasterised to desired glyph pixel size

splines

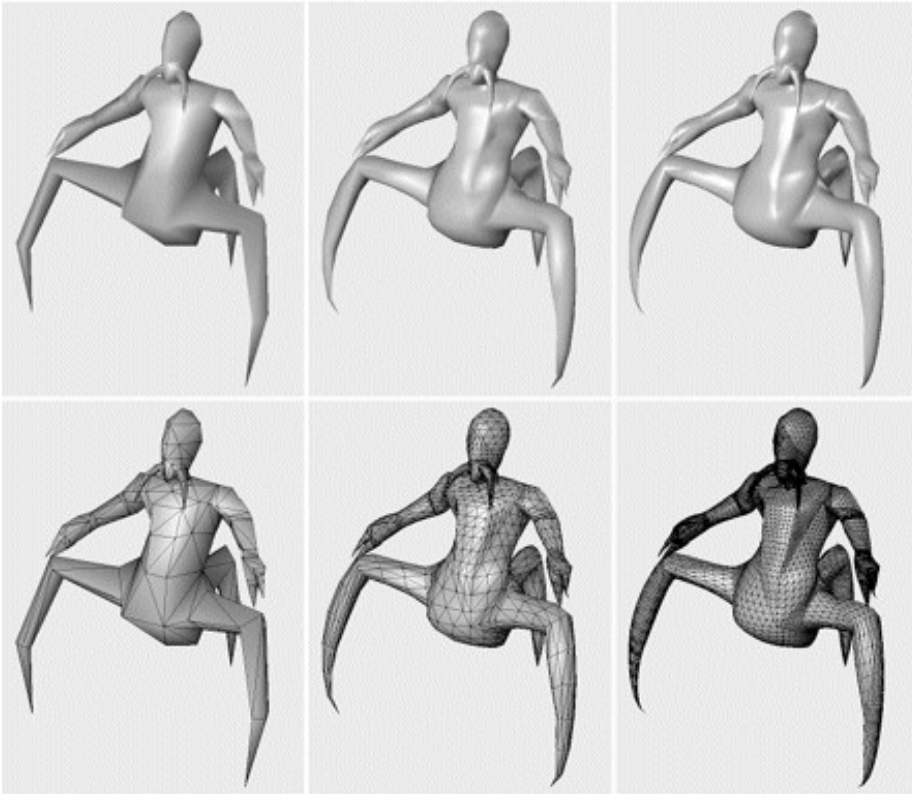
- Long curve made up of several curves (of any type)
- Start and end control points called “welds”
- In-between control points called “knots”
- If “t” is...
 - between 0 and 1 = 0..1 for first curve
 - Between 1 and 2 = becomes 0..1 for second curve
 - etc.

Hermite curves (and splines)

- Start and end points have a velocity
 - Indicates curve direction
- Can chain any number of points together



coolest modern use of curves: tessellation with Bézier triangles



- Level of Detail (LOD)
- Control points at vertices
- More triangles generated
- Curve equation
- Tessellation shaders

Source: id software. Appears in Gamasutra and Real-Time Rendering book

Tmrw: exam revision

**Mon: final lecture
(any requests?)**

Tue: demos! [2 hrs]